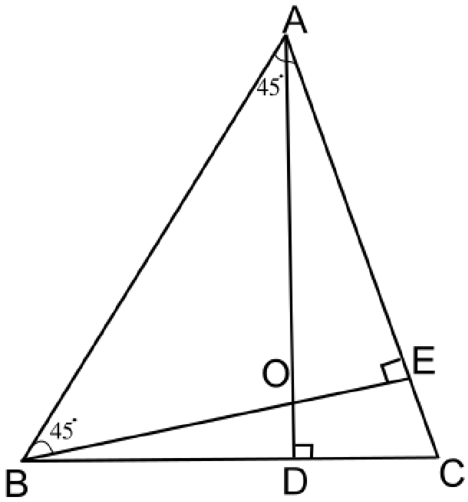
Author's Solution

Before going to the solution, let us see an important Geometric result.

Geometric result 1: (45°Theorem):

InΔ ABC, if $\angle A = 45°$, and if 'O' is its orthocentre, then AO= BC.



Proof :

Construction :

Draw the altitudes AD & BE.

Now, $\angle ABE = \angle BAE = 45° \Rightarrow$ AE = BE -----------------------------------(1)

In ΔAOE & ΔBCE

(1) $\longrightarrow$ AE=BE

$\angle AEO = \angle BEC = 90°$

And $\angle EBC = \angle EAO$ $\quad (\because AEDB \text{ is concyclic})$

$\therefore \Delta AOE \cong \Delta BCE$ **(AAS Principle)**

$\therefore AO = BC$ **----------------------- Proved.**

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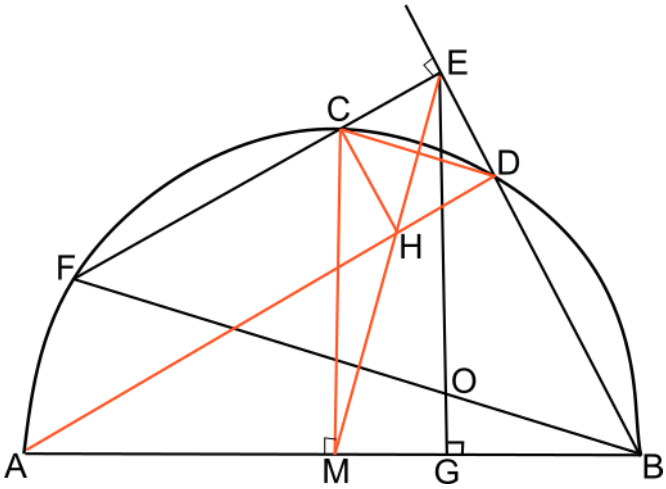
Now, let us go to the solution of this month's (NOV-23) question.

Given :

ACB is a semicircle with 'C' as its midpoint. D is a point on arc BC. CE$\perp$ BD produced. EC produced meets the semi circle at F. EG$\perp$ AB. EG & FB meet at O.

To prove:

EO = $\frac{1}{2}AB$



Construction :

Mark the centre of AB at M. Join CM, AD, EM & CD. Let AD & EM meet at H. Join CH.

Solution :

C is the midpoint of semicircle AB & M is the midpoint of the diameter AB. $\therefore CM \perp AB$.

'C' is a point on the circumcircle of ΔADB and CE & CM are perpendicular lines drawn from 'C' to its sides BD & AB.

$\therefore EM$is the Simson's line of ΔABD

$\therefore$ CH$\perp$AD. (Third side)

Now, In Quadrilateral CEDH,

$\angle CHD = 90°$

$\angle HDE = 90°$ **(Angle in the semicircle)**

$\angle CED = 90°$ **(Given)**

$\therefore \angle ECH = 90°$

$\Longrightarrow$ **CEDH is concyclic.**

$\angle BFC = 45°$ **(Angle borne by an arc BC which is $\frac{1}{4}$ of the circle)**

$\Longrightarrow \angle ECD = 45°$ ($\because$ *CFBD is concyclic*) **------------------(1)**

$\Longrightarrow \angle DCH = 45° = \angle HED$ **(Note that here CHDE is a square).**

$\Longrightarrow \angle MEB = 45°$

$\therefore \angle EBF = 90° - \angle EFB = 90° - 45° = 45°$**-------------------------(2)**

(1)$\longrightarrow \angle HED = \angle CEH = 45°$

$\Longrightarrow FB \perp EM$

$\Longrightarrow$ **O is the orthocentre of ΔMBE**

(2) $\longrightarrow \angle MEB = 45°$

$\therefore$ **As per $45°$ Theorem given above**

EO= MB=$\frac{1}{2}AB$ --------------------------------------Proved

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